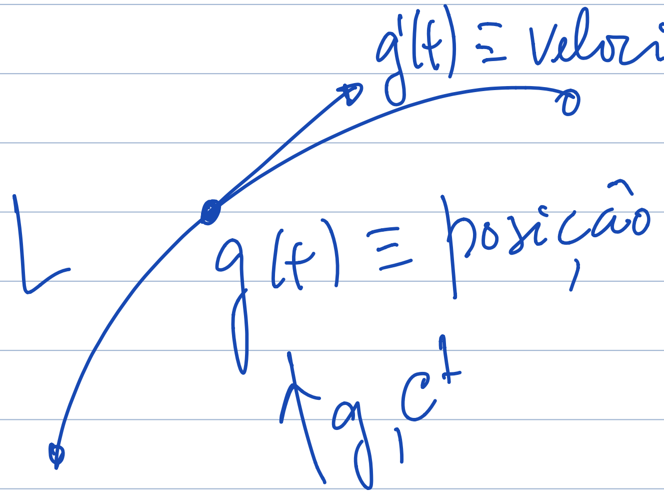
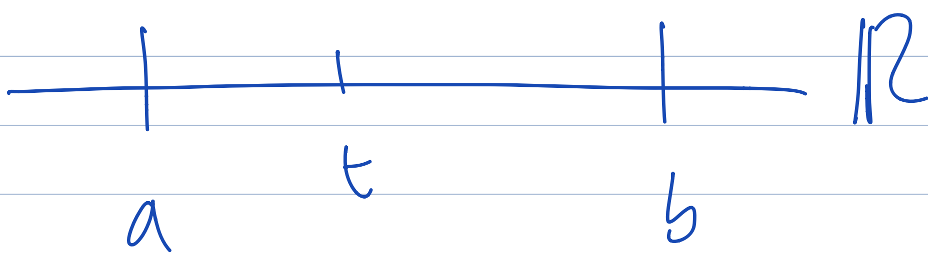
CDI-II - Prática 18/5/21

Ficha 10: $\varphi : \mathbb{R}^n \to \mathbb{R}$

L linha em $\mathbb{R}^n$.



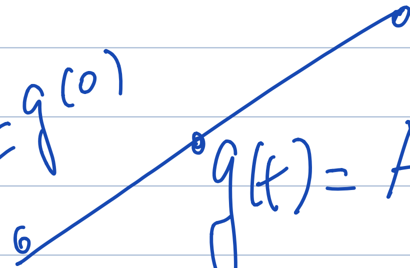
$\|g'(t)\| \equiv$ velocidade escalar



$$\int_L \varphi = \int_a^b \varphi(g(t))\,\|g'(t)\|\,dt$$

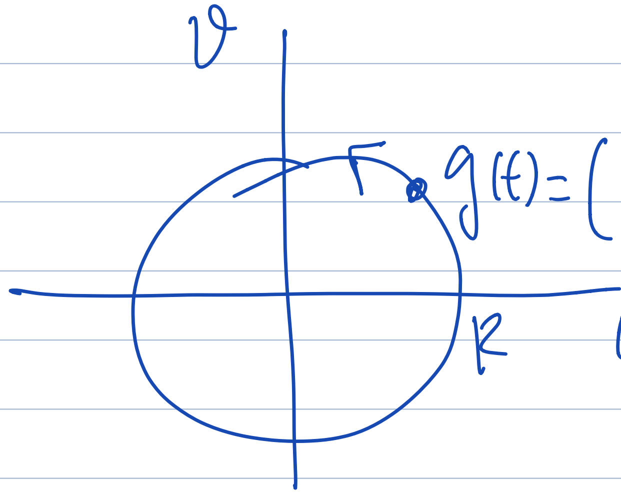
$B = g(1)$

$A = g(0)$



$$g(t) = A + t(B - A)$$

$$0 \le t \le 1.$$

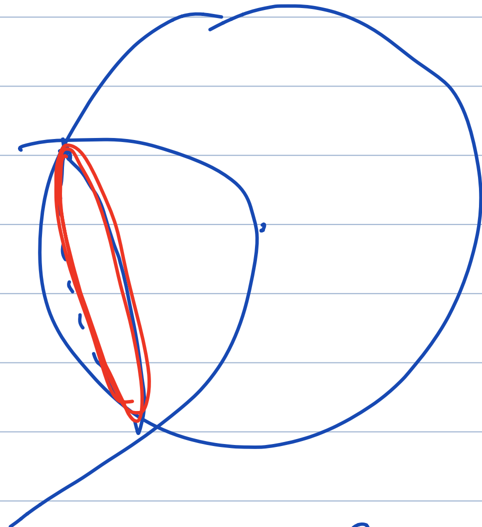


$$u^2 + v^2 = R^2$$

$$g(t) = (R\cos t, R\sin t)$$

$$0 \le t \le 2\pi$$

$$6 - L : \begin{cases} x = y^2 + z^2 \\ x^2 + y^2 + z^2 = 2 \end{cases}$$



$$\begin{cases} x = y^2 + z^2 \geq 0 \\ x^2 + x - 2 = 0 \end{cases} \qquad \cancel{x = -2} \qquad \boxed{x = 1}$$

$$\begin{cases} y^2 + z^2 = 1 \\ x = 1 \end{cases} \qquad \begin{array}{l} y = \cos t \\ z = \operatorname{sen} t \\ \boxed{x = 1} \end{array}$$

$$g(t) = (1, \cos t, \operatorname{sen} t), \quad 0 \leq t \leq 2\pi$$

Centróide: $(\bar{x}, \bar{y}, \bar{z})$

$$\bar{x} = \frac{\int_L x}{\int_L 1} \quad \begin{array}{l} \longrightarrow \varphi(x,y,z) = x \\ \longrightarrow \varphi(x,y,z) = 1 \end{array}$$

$x = 1$ em $L \Rightarrow \boxed{\bar{x} = 1}$

$$\bar{y} = \frac{\int_L y}{\int_L 1} \quad \begin{array}{l} \longrightarrow \varphi(x,y,z) = y \\ \longrightarrow \text{Comprimento de } L = 2\pi \end{array}$$

$$\int_L y$$

$g'(t) = (0, -\operatorname{sen} t, \cos t)$

$\|g'(t)\| = \sqrt{\operatorname{sen}^2 t + \cos^2 t} = 1$

$$\int_L y = \int_0^{2\pi} \cos t \, dt = 0.$$

$$\bar{y} = \frac{0}{2\pi} = 0.$$

$$\int_L z = \int_0^{2\pi} \operatorname{sen} t \, dt = 0$$

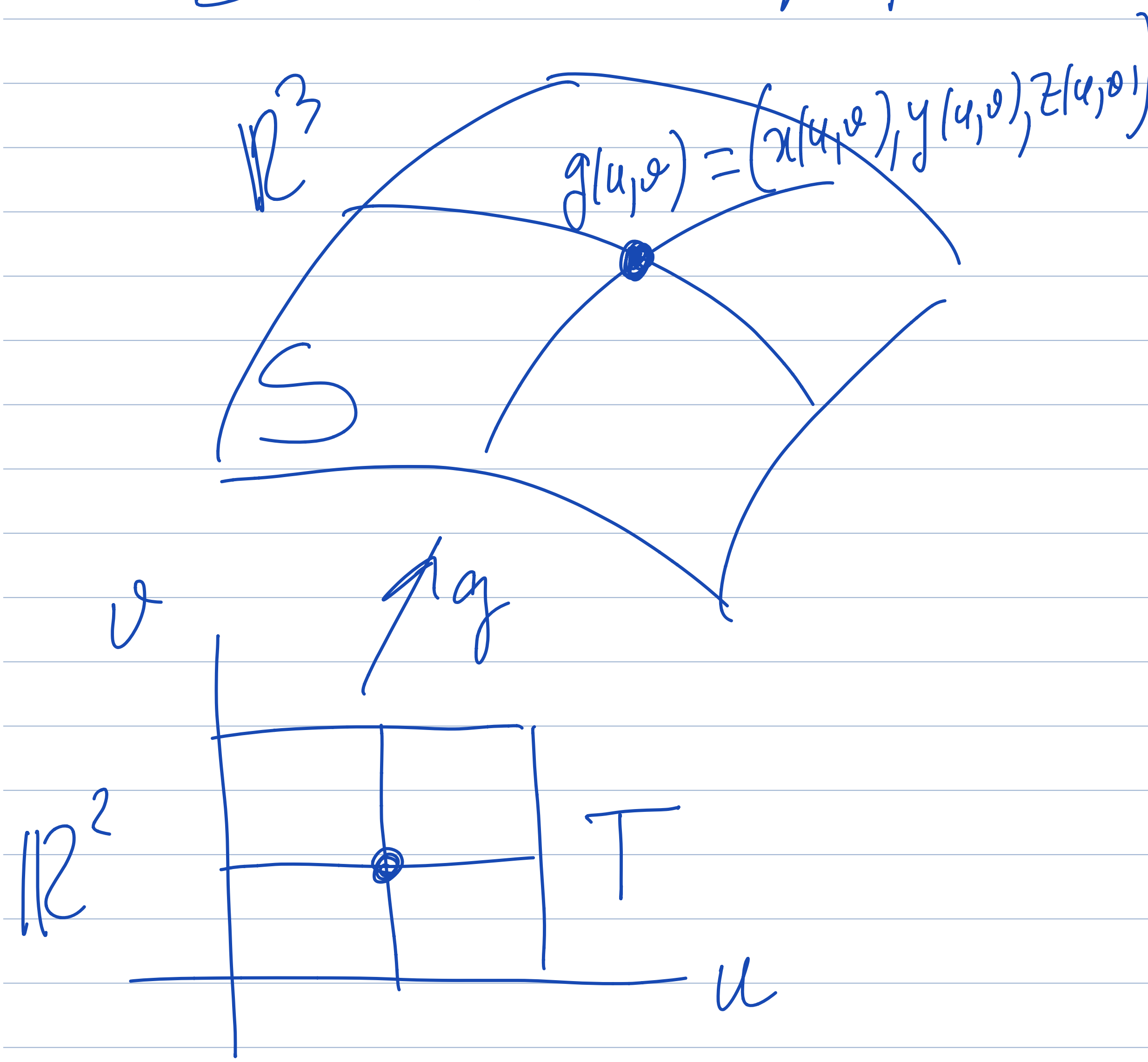
$$\bar{z} = \frac{0}{2\pi} = 0$$

$(1, 0, 0)$ centroide.

Superficies en $\mathbb{R}^3$.

$\varphi : \mathbb{R}^2 \longrightarrow \mathbb{R}$

$S \subset \mathbb{R}^3$ superficie



$$\int_S \varphi = \iint_T \varphi(g(u,v)) \sqrt{\det Dg^T Dg}\, du\, dv$$

7-a) $A: \quad 1+\sqrt{x^2+z^2} = y$

(x,y,z) ↺

$y<2$

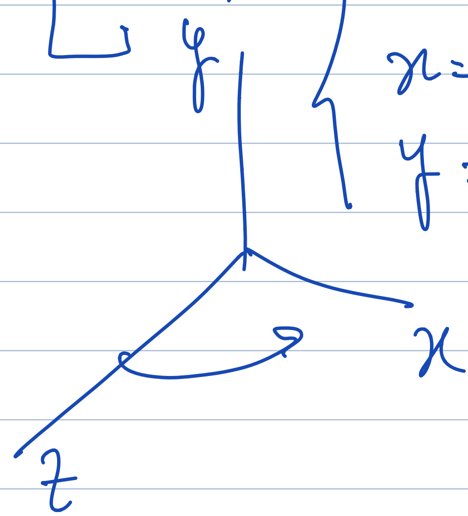
$\boxed{x>0}$

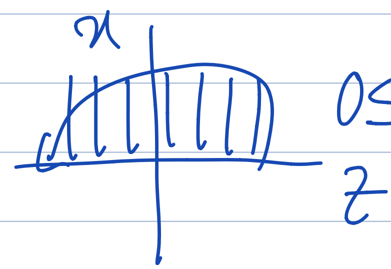
$\rho = \sqrt{x^2+z^2}$

(ρ, θ, y)

$$\begin{cases} z = \rho\cos\theta \\ x = \rho \operatorname{sen}\theta \\ y = y \end{cases}$$

$\boxed{1+\rho = y}$





$0 \le \theta \le \pi$

$$g(\rho,\theta) = \left(x(\rho,\theta), y(\rho,\theta), z(\rho,\theta)\right)$$

$$T: \begin{cases} 0 \leq \theta \leq \pi \\ 0 < \rho < 1 \end{cases}$$

$0 \leq \rho = y - 1$

$< 2 - 1 = 1$

$$Dg^T Dg = \begin{bmatrix} \|T_1\|^2 & T_1 \cdot T_2 \\ T_2 \cdot T_1 & \|T_2\|^2 \end{bmatrix}_{2\times 2}$$

simétrica

$$Dg(\rho,\theta) = \begin{bmatrix} \vdots & \vdots \\ T_1 & T_2 \\ \vdots & \vdots \end{bmatrix}_{3\times 2}$$

$T_1 = D_\rho g$

$T_2 = D_\theta g$

$$g(\rho,\theta) = (\rho \operatorname{sen}\theta,\ 1+\rho,\ \rho\cos\theta)$$

$$T: \begin{cases} 0 < \rho < 1 \\ 0 < \theta < \pi \end{cases}$$

$$T_1 = D_\rho g = (\operatorname{sen}\theta,\ 1,\ \cos\theta)$$

$$T_2 = D_\theta g = (+\rho\cos\theta,\ 0,\ -\rho \operatorname{sen}\theta)$$

$$Dg^T Dg = \begin{bmatrix} 2 & 0 \\ 0 & \rho^2 \end{bmatrix}_{2\times 2}$$

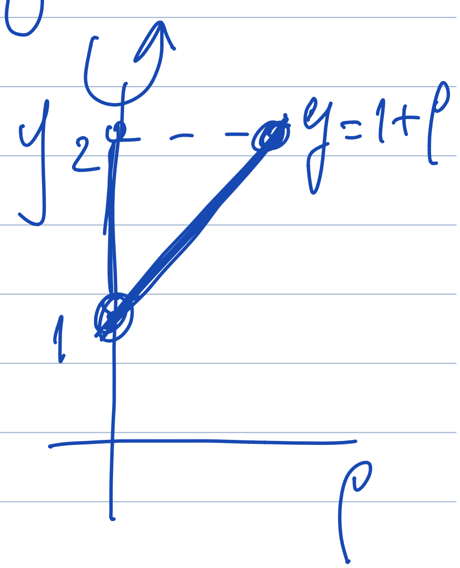
$$\sqrt{\det Dg^T Dg} = \sqrt{2\rho^2} = \sqrt{2}\,\rho$$

$$\mathrm{Vol}_2(A) = \int_A 1 =$$

$$= \iint_T \sqrt{2}\,\rho \, d\rho \, d\theta$$

$$= \sqrt{2} \int_0^{\pi} \left(\int_0^1 \rho \, d\rho \right) d\theta$$

$$= \sqrt{2}\, \pi/2 \, . \; //$$



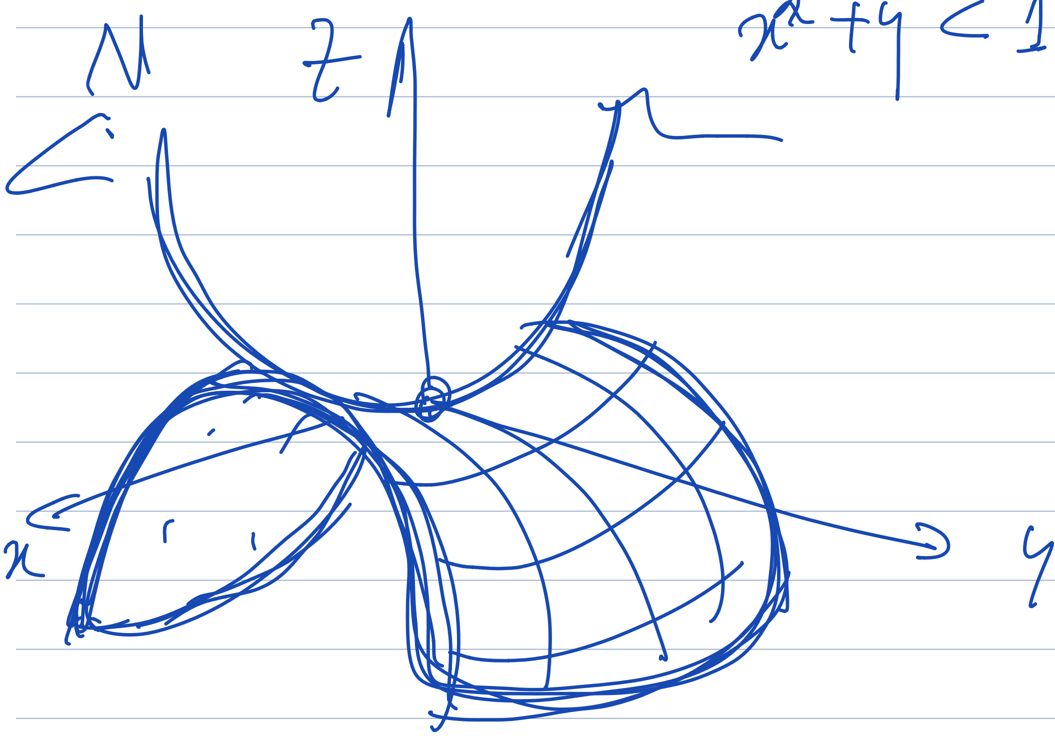
$$g(\theta, y) = \big((y-1)\sin\theta,\ y,\ (y-1)\cos\theta\big)$$

$\rho = y - 1$ $\qquad T: \begin{cases} 1 < y < 2 \\ 0 < \theta < \pi \end{cases}$

etc ...

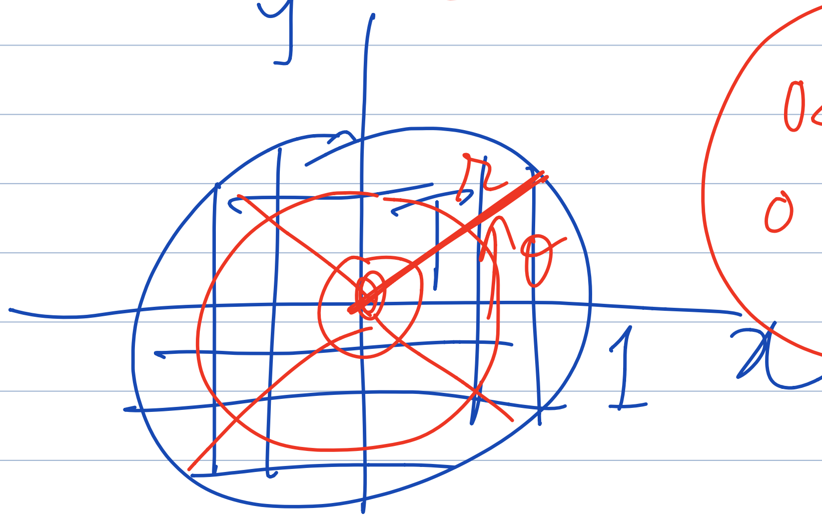
7-b) B: $\boxed{z = xy}$

$x^2 + y^2 < 1$



$$g(x,y) = (x, y, xy)$$

$$T: \; x^2 + y^2 < 1$$



$$0 < \theta < 2\pi$$
$$0 < r < 1$$

$$D_x g = (1, 0, y)$$

$$D_y g = (0, 1, x)$$

$$\det Dg^T Dg = \det \begin{bmatrix} 1+y^2 & xy \\ xy & 1+x^2 \end{bmatrix} =$$

$$= (1+x^2)(1+y^2) - x^2y^2$$

$$= 1 + y^2 + x^2 + \cancel{x^2y^2} - \cancel{x^2y^2}$$

$$= 1 + x^2 + y^2$$

$$\sqrt{\det Dg^T Dg} = \sqrt{1+x^2+y^2}$$

$$\mathrm{vol}_2(B) = \iint \sqrt{1+x^2+y^2}\, dx\, dy$$

T

Mudar para (r, θ) : $\boxed{r\,dr\,d\theta}$

$$\mathrm{vol}_2(B) = \int_0^{2\pi}\int_0^1 \sqrt{1+r^2}\; r\,dr\,d\theta$$

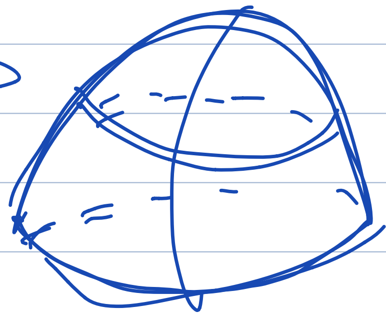
etc...

8 - S : $\boxed{x^2+y^2+z^2=a^2}$, $a>0$

$z \geq 0$

$\boxed{\rho = a}$

S



$$\int_S f(x,y,z)$$

$f(x,y,z) = x^2+y^2$

(ρ, θ, φ), $\rho = a$

$$\left.\begin{array}{l} 0<\varphi<\frac{\pi}{2} \\ 0<\theta<2\pi \end{array}\right\} T$$

$g(\theta,\varphi) = \big(x(\theta,\varphi), y(\theta,\varphi), z(\theta,\varphi)\big)$

$g(\theta,\varphi) = \big(\underbrace{a \operatorname{sen}\varphi}\cos\theta, \underbrace{a\operatorname{sen}\varphi}\operatorname{sen}\theta, a\cos\varphi\big)$

$$\sqrt{\det Dg^T Dg} = a^2 \operatorname{sen}\varphi$$

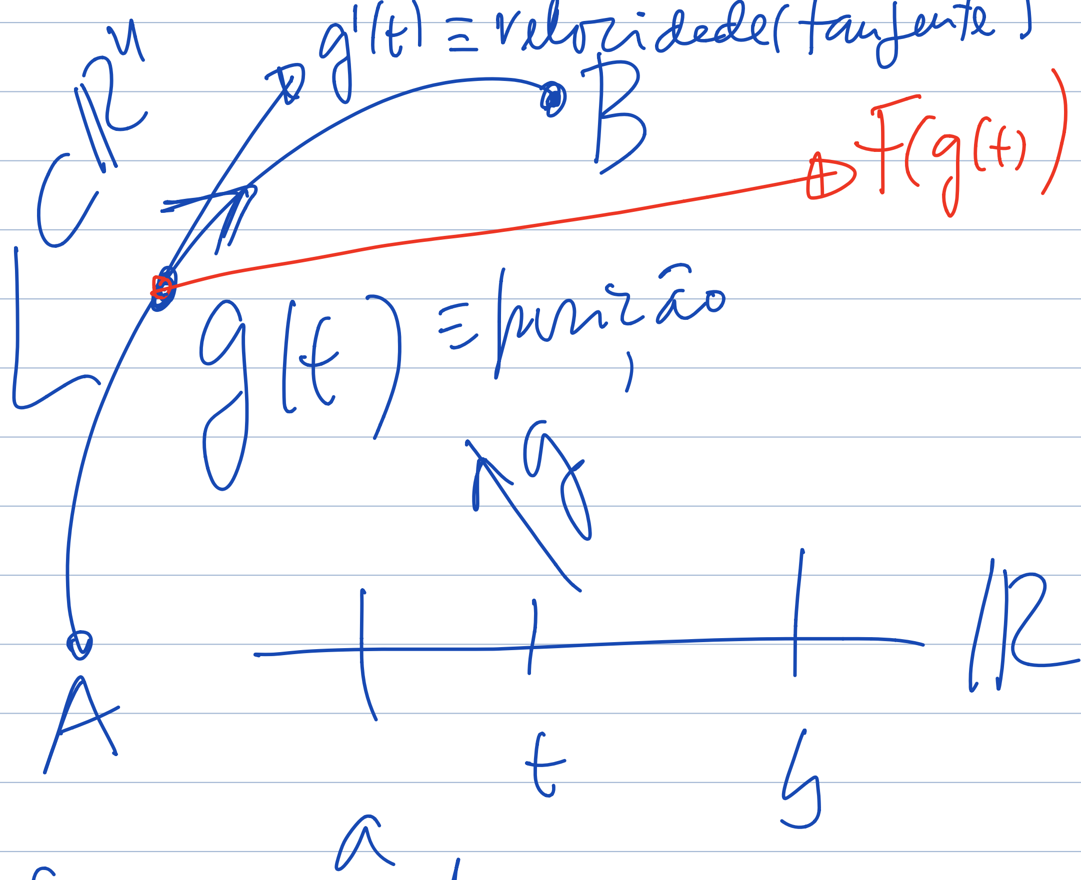
$$I_z(S) = \iint_T a^2 \operatorname{sen}\varphi \; a^2 \operatorname{sen}^2\varphi \, d\theta \, d\varphi$$

$$= a^4 \int_0^{2\pi} \left(\int_0^{\pi/2} \operatorname{sen}\varphi \operatorname{sen}^2\varphi \, d\varphi \right) d\theta$$

$$= 2\pi a^4 \int_0^{\pi/2} \operatorname{sen}\varphi \left(1-\cos^2\varphi\right) d\varphi$$

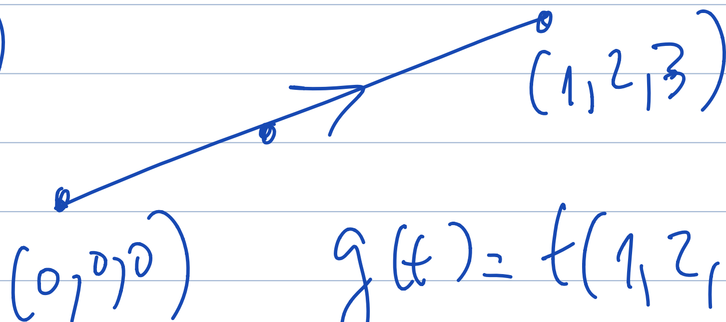
etc . . .

Ficha 12: Trabalho realizado por um campo de forças ao longo de uma linha (Integral de linha de um campo vectorial)



$$W = \int_L F \cdot dg = \int_a^b F(g(t)) \cdot g'(t)\, dt$$

2 - a)



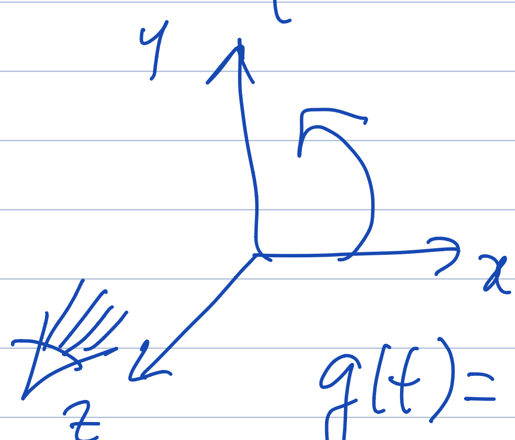
$$g(t) = t(1,2,3) = (t, 2t, 3t)$$

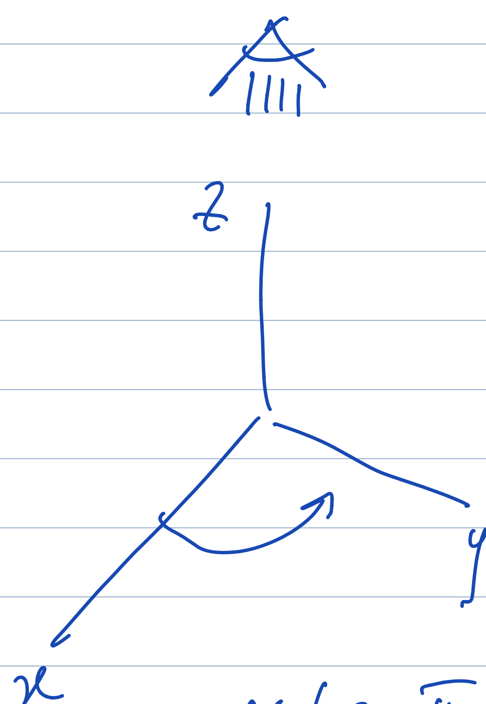
$$0 \leq t \leq 1.$$

$$g'(t) = (1,2,3)$$

etc.

2 - b) $\begin{cases} x^2 + y^2 = 1 \\ z = x^2 - y^2 \end{cases}$





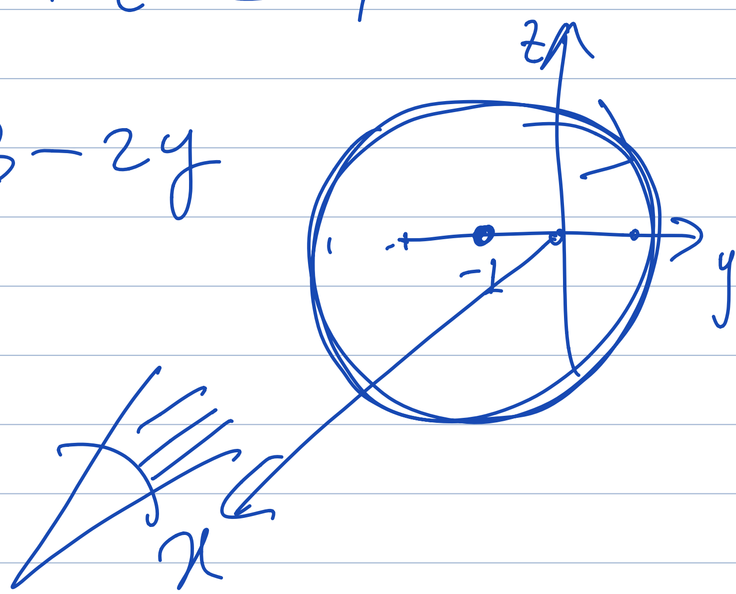
$0 \leq t \leq 2\pi$

$$g(t) = (\cos t, \operatorname{sen} t, \cos^2 t - \operatorname{sen}^2 t)$$

2-c) $\begin{cases} x = y^2 + z^2 \\ 2y + x = 3 \end{cases} \Leftrightarrow \begin{cases} 3 - 2y = y^2 + z^2 \\ x = 3 - 2y \end{cases}$

$\Leftrightarrow \begin{cases} \overline{y^2 + 2y + 1} + z^2 = 3 + 1 \\ x = 3 - 2y \end{cases}$

$\Leftrightarrow \begin{cases} (y+1)^2 + z^2 = 4 \\ x = 3 - 2y \end{cases}$



$y + 1 = 2\cos t \qquad \begin{cases} y = 2\cos t - 1 \\ z = -2\sin t \end{cases}$

$z = -2\sin t$

$x = 3 - 2(2\cos t - 1)$ $\boxed{0 \leq t \leq 2\pi}$

$g(t) = (5 - 4\cos t, 2\cos t - 1, -2\sin t)$

etc..

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